A Relation Between Twin Primes and Their Indices

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A careful study of the Quran's 19-based mathematical structure served as the genesis for discovering a relation between twin primes and their indices [1]. As we know there is an asymptotic relation discovered by Gauss between primes and their indices. However, no analytic relation has been proposed for twin primes and their respective indices. Our relation for twin primes distributions also coincides with the distribution of a set numbers which we refer to as twin prime's companion or TPC for short. We have utilized these numbers to generate twin primes as a function of their indices. A relation of the following form seems to reproduce the general trend of twin primes distribution.

(1)
$$i \ln i = \frac{x}{\ln x}$$

The problem of dealing with prime numbers have always been a source of fascination with mathematicians since numbers were invented. We know very little about this subset of natural numbers which possess no divisor except unity and themselves. We know for example, that there are infinitely many primes. In the 19th century Gauss discovered "the inte-

gral logarithm of x."

$$(2) \qquad Li(x) = \int_2^x \frac{dt}{\ln t}$$

By 1900 mathematicians inspired by the work of Riemann developed and proved an asymptotic relation which is known today as "the prime number theorem."

$$(3) \qquad \lim_{x \to \infty} \frac{\pi(x)}{x/\ln x} = 1$$

This was a monumental discovery since for the first time an analytic "fit" was relating primes to their indices.

A subset of primes, known as twin primes, are even more fascinating. These primes differ only by two in magnitude and it is not clear if there are infinitely many twin primes. In this note, we discuss with the aide of a few plots a possible analytic relation for the distribution of twin primes. Before proceeding further, let us discuss a set of numbers, which we have called TPC. The TPC's are a subset of composites bracketed by twin primes. The first ten TPC's are: 4, 6, 12, 18, 30, 42, 60, 72, 102, and 108. Naturally, the index of TPC and the twin primes are the same. These composite have a few properties which one can observe. These properties, although not unique, however, could be helpful in locating twin primes. All TPC's are a multiple of 6, except the first TPC which is 4. All TPC's except for the first two end up with 0, 2, or 8.

We have written a program to generate the first 1,400,000 TPC's. This index corresponds to a TPC with a magnitude of

368116602. The algorithm generates numbers that are multiples of 6 and eliminates the ones that are ending with 4 and 6. The program then checks the two, before and after numbers for primality. The plots shown in Figs. 1 and 2 are displaying TPC's as a function of their indices. These plots are clearly indicative of an asymptotic behaviour of twin primes or TPC's distribution. Shown in Fig. 3 is a comparison of the actual TPC/LOG(TPC) as a function of their indices (solid circles) and the prediction of our analytic expression of Eqn. 1 (dashed line). The relation we have suggested in this note for the twin primes and their indices may not be unique; however, it gives a good approximation for the distribution of TPC's. For possible asymptotic behaviour of TPC's and of Eqn. 1, comparison at much larger values of indices is desirable. Obviously, more work is required.

In summary, we have suggested a relation between twin primes and their indices (Eqn. 1). We have explained the role of a set of numbers bracketed by twin primes which we have called twin prime's companions. Because of their properties, these numbers were instrumental in generating twin primes.

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Figure Captions

1. A linear plot of TPC vs. index. The highest index is 1.4×10^6.

2. A semi-log plot of the TPC vs. index. Same points as Fig. 1.

3. A semi-log plot of the TPC/LOG(TPC) vs. index. Solid circles represent the actual TPC's and the dashed line is the prediction of Eqn. 1.

Reference

1. Quran, chapter 2, verse 1.

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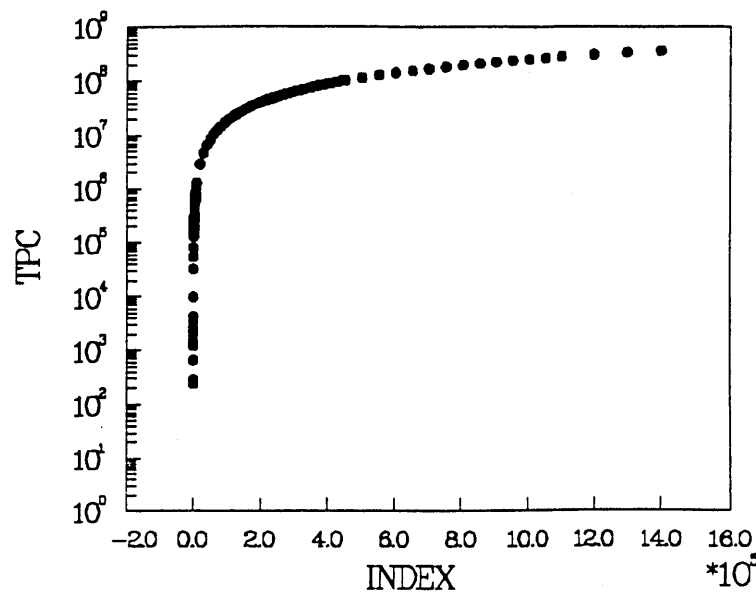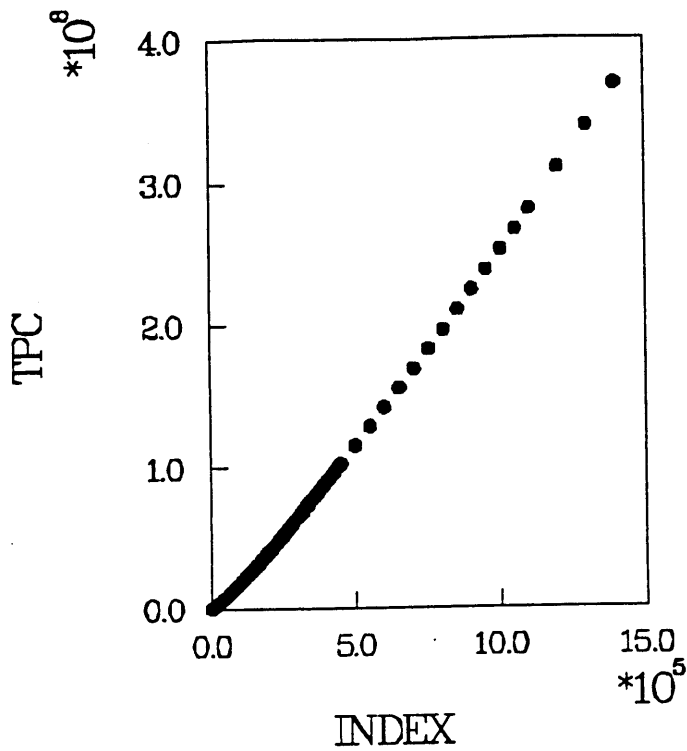*Prairie View A and M University,*
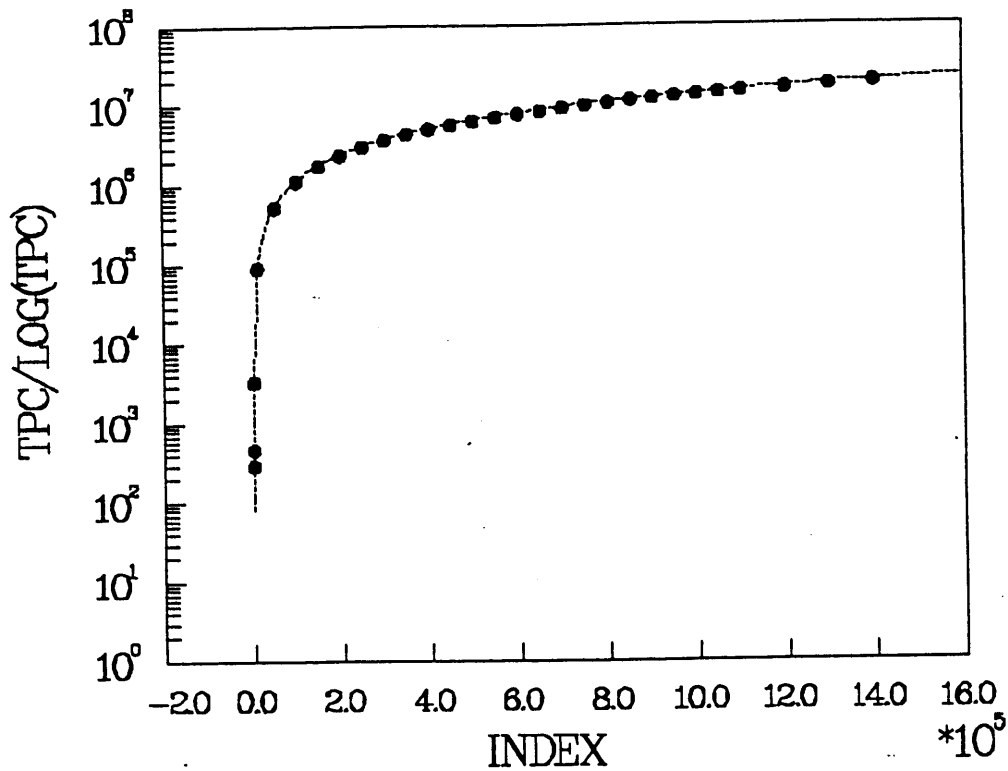*Prairie View, TX 77446*

Fig. 1

Fig. 2

Fig. 3